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# Misc

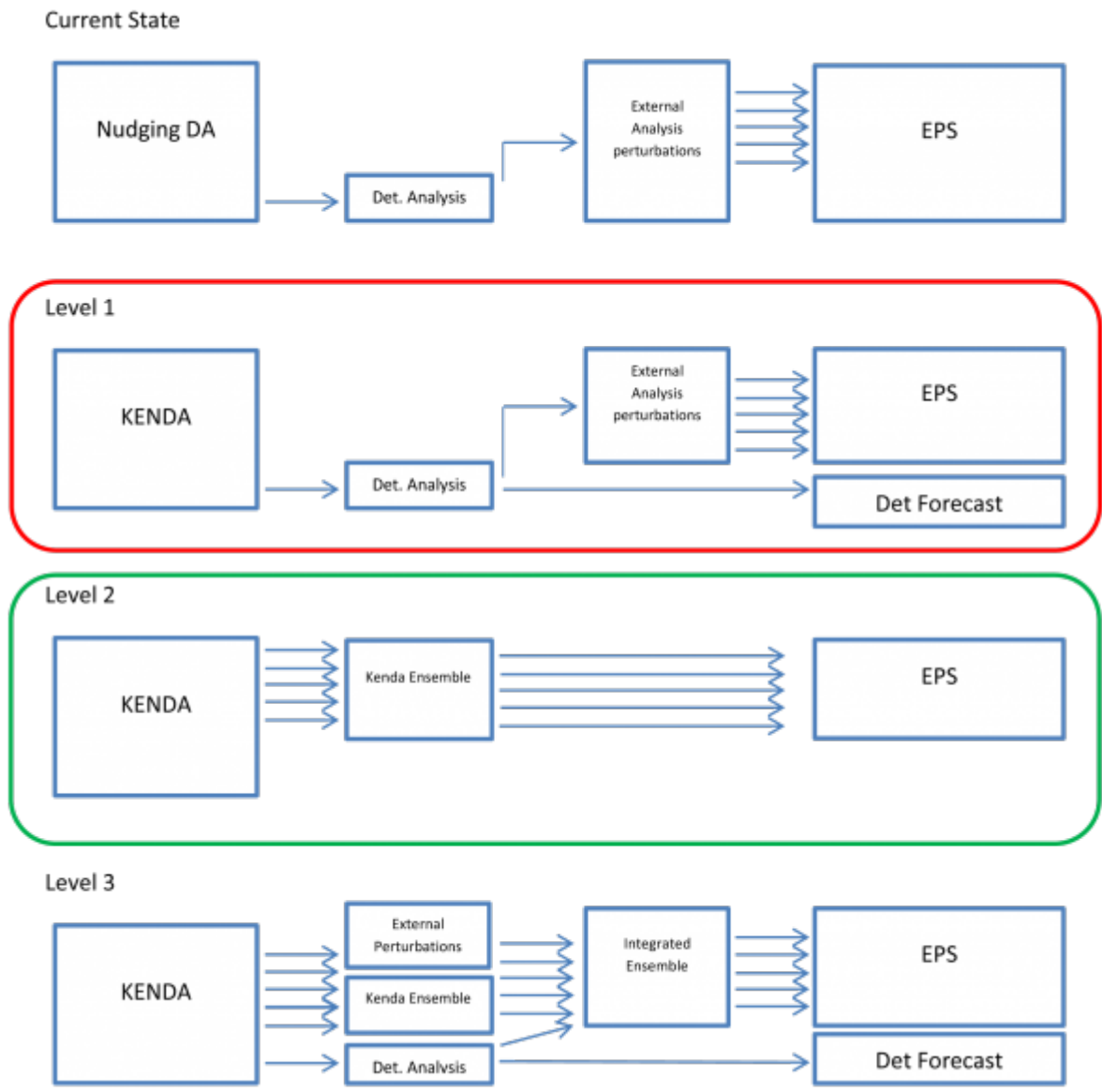
This is a page for various notes and quick tests ...

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Many times I had someone on the other part of the line and
wanted to talk about mathematics ... so you need some space
where you can quickly type some ideas in LaTeX and the other
person with internet connection can read it ...
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## Talk Data Assimilation

A general introductory talk into data assimilation for numerical weather prediction:  
[http://scienceatlas.com/potthast/\\_media/talk\\_potthast\\_reading\\_2014\\_11.pdf](http://scienceatlas.com/potthast/_media/talk_potthast_reading_2014_11.pdf) and  
[http://scienceatlas.com/potthast/\\_media/potthast\\_icon\\_bacy\\_tour.pdf](http://scienceatlas.com/potthast/_media/potthast_icon_bacy_tour.pdf)

## KENDA Concept



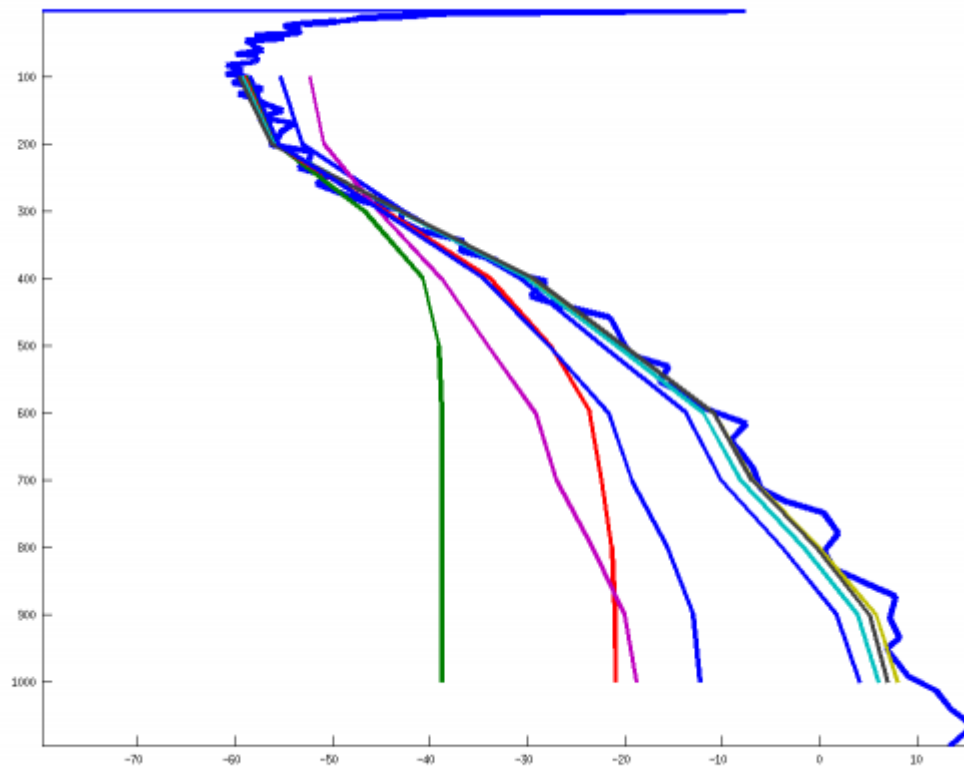
## Approximations in Fourier Space

Aamir August 30. We try to approximate given function values  $f(x_1), f(x_2), f(x_3), \dots, f(x_m)$  by a function of the form 
$$g(x) = \sum_{|\ell|=-L}^L g_{|\ell|} e^{i|\ell|x}, \quad x \in [0, 2\pi].$$
 We do it

1. globally with large  $L$ ,
2. globally with small  $L$  (i.e. low modes)
3. locally with small  $L$ , this is called `Localization`.

## SEVIRI Testing

The following graphics shows some testing with RTTOV 10 and SEVIRI Radiances. The blue profile is the demo temperature profile provided by RTTOV. The other lines show the brightness temperature (BT) of SEVIRI (in degrees Celsius) given by the channels when an opaque cloud is put into the different heights (in hPa) shown on the left.



## Test PHP

Embedding PHP works, writing "test" via php: test

**Natalie June 10, 2013**

### I) Theory for the Stability of Classification in General

... we investigated the stability of image classes under the influence of the above compact operator  $A$ . Our main results show that

- the image classes of two stably separable linear classes will not be stably separable if the normal vector to the separating hyperplane is not in the range of the adjoint operator  $A^*$ .
- For sequences of uniformly stably separable linear classes, the stability of the separation is not preserved when we map them into the image space.
- For two elliptical classes, in any case the image classes remain stably separable.

We note that the second point has consequences for non-linear classes as well, since in the smooth case they are made up of sequences of linear classes. If the nonlinear class has a sequence of normal vectors which span an infinite dimensional subspace, then the nonlinear image classes cannot be stably separable.

### II) Investigate the Stability of Key Algorithms

We have investigated two well-known algorithms for classification: the FLD as a key method for supervised classification and the K-Means Algorithm combined with a PCA as a well established approach to unsupervised classification.

- For FLD, by tools of numerical analysis we have proven that - applied to image data - the classes inherit the ill-posedness of the mapping  $A$ .

### III) Investigate the Stability of Classifications in Magnetic Tomography (in particular for Fuel Cells)

In Chapter 3, we presented a concise investigation of the ill-posedness of classifications for magnetic tomography, with a particular emphasis on classifications for fuel-cells as an important application area.

## Natalie June 6, 2013

Hello 😊

Statement: Let  $A: X \rightarrow Y$  be a compact operator and  $X$  be infinite dimensional. Then there is a sequence  $\{\varphi_n \in X\}$  with  $\|\varphi_n\|=1$  such that 
$$A\varphi_n \rightarrow 0, \quad n \rightarrow \infty.$$

*Proof.* Since  $A^{-1}$  is unbounded, this means that there exists a sequence  $\{\psi_n \in Y\}$  with norm  $\|\psi_n\|=1$  and 
$$\|A^{-1}\psi_n\| \rightarrow \infty, \quad n \rightarrow \infty.$$
 We define 
$$\varphi_n := \frac{A^{-1}\psi_n}{\|A^{-1}\psi_n\|}, \quad n \in \mathbb{N}.$$
 Then we have  $\|A\varphi_n\| = \frac{\|\psi_n\|}{\|A^{-1}\psi_n\|} \rightarrow 0, \quad n \rightarrow \infty,$  and the proof is complete  $\square$ .

## Mathematical Notation Discussion

Use  $x$  for points in  $\mathbb{R}^3$ , then 
$$\Omega = \bigcup_{i=1}^K C_i$$

probably better to write 
$$\Omega = \bigcup_{C \in \mathcal{C}} C$$
 where 
$$\mathcal{C} = \bigcup_{j=1}^K L_j$$

Consider a simple situation like

$x_1$	$x_2$
$x_3$	$x_4$

and now two slants, given by 
$$\begin{matrix} s_1 & = & x_1 + x_4 \\ s_2 & = & x_2 + x_4 \end{matrix}$$
 leading to a matrix equation 
$$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} s_1 \\ s_2 \end{pmatrix}$$
 We might include some weighting function for taking care of the length of the path of the ray through the area denoted by  $x_j$ ,  $j=1, \dots, 4$ , then we obtain a weighted system of the form 
$$\begin{matrix} s_1 & = & w_{1,1} x_1 + w_{1,4} x_4 \\ s_2 & = & w_{2,2} x_2 + w_{2,4} x_4 \end{matrix}$$

A ray starting at  $b \in \mathbb{R}^3$  with direction  $\nu \in \mathbb{R}^3$  is given by 
$$L = \{ x = b + \eta \cdot \nu \mid \eta > 0 \}.$$
 An integral along  $L$  is written as 
$$f(b, \nu) := \int_L \varphi(y) ds(y),$$
 where  $ds(y)$  denotes the standard Euclidean measure along  $L$ . Alternatively, when  $\|\nu\| = 1$ , you might write 
$$f(b, \nu) = \int_0^\infty \varphi(b + \eta \nu) d\eta$$
 We use the notation 
$$(R\varphi)(b, \nu) := f(b, \nu), \quad b \in \mathcal{B}, \quad \nu \in \mathcal{N}$$
 where  $\mathcal{B}$  and  $\mathcal{N}$  denote the set of our base stations and the set of slant directions measured by the stations over some time interval. In discretized form, the operator  $\mathbf{H}$  maps  $\vec{\varphi} \in \mathbb{R}^n$  into  $\vec{f} \in \mathbb{R}^m$ . It is a linear operator and, thus, we obtain a matrix operator from  $\mathbb{R}^n$  into  $\mathbb{R}^m$ . Thus, GPS tomography boils down to solving a linear finite dimensional system 
$$\mathbf{H} \vec{\varphi} = \vec{f}$$
 We solve it by 3dVar or Tikhonov Regularization, respectively, which is 
$$\varphi^{(a)} = \varphi^{(b)} + B^T (R + H B H^T)^{-1} (f - H \varphi^{(b)})$$
 which is minimizing 
$$J(\varphi) = \|\varphi - \varphi^{(b)}\|_{B^{-1}}^2 + \|f - H \varphi\|_{R^{-1}}^2.$$
 Adaptive regularization would change the regularization term 
$$\|\varphi - \varphi^{(b)}\|_{B^{-1}}^2$$

## Natalie 24.4.2013

We first select a subsequence of  $\mathbb{N}$  such that a point in the support  $\chi_i$  of  $v_{k_i}$  has limiting point  $x_{\ast} \in \Omega$ . This is possible since any bounded sequence in  $\mathbb{R}$  has a convergent subsequence. We call it  $k_i$  again.

Given  $\epsilon > 0$  there is  $i > 0$  such that  $\|v_{k_i} - v_{\ast}\| < \epsilon$ . So we know that on the exterior of the support  $\Omega_{k_i}$  of  $v_{k_i}$  we have  $\|v_{\ast}\|_{L^2(\Omega \setminus \Omega_{k_i})} < \epsilon$  For a function  $v_{\ast} \in L^2(\Omega)$  we have that 
$$\left| \int_{\Omega_{k_i}} |v_{\ast}(y)|^2 dy \right| \rightarrow 0, \quad i \rightarrow \infty.$$
 Now, we obtain 
$$\|v_{\ast}\|_{L^2(\Omega)}^2 = \|v_{\ast}\|_{L^2(\Omega \setminus \Omega_{k_i})}^2 + \|v_{\ast}\|_{L^2(\Omega_{k_i})}^2 \rightarrow 0, \quad i \rightarrow \infty.$$

## Natalie 9.5.2013

Simple question: is  $\frac{1}{x-y}$  analytic in  $x$  for fixed  $y$ ?

Answer 1) Yes, since the sum and product of analytic functions are analytic, and the quotient of analytic functions are analytic where the denominator is non-zero.

Answer 2) We use the series expansion  $\frac{1}{x-y} = (-1) \sum_{n=1}^\infty \frac{x^n}{y^{n+1}}$  for  $|x| < |y|$ , which I took from [Wolfram](#). For  $|x| < \rho \cdot |y|$  with  $\rho < 1$  we have  $\left| \frac{x^n}{y^{n+1}} \right| < \frac{1}{|y|} \rho^n$ , such that  $\sum_{n=1}^\infty \frac{x^n}{y^{n+1}} < \frac{1}{|y|} \sum_{n=1}^\infty \rho^n$  is absolutely convergent by the geometric series. Thus,  $\frac{1}{x-y}$  is analytic on  $|x| < |y|$ . For  $|x| > |y|$  we use  $\frac{1}{x-y} = \sum_{n=1}^\infty \frac{y^n}{x^{n+1}}$  analogously.

## little statement

Let  $g$  be a function in  $L^2(\Omega)$  and let  $\Omega_i$  be subsets of  $\Omega$  with  $|\Omega_i| \rightarrow 0$  for  $i \rightarrow \infty$ . Then we have 
$$\int_{\Omega_i} |g|^2 dy \rightarrow 0, \quad i \rightarrow \infty.$$

*Proof.* Given  $\epsilon$  there is a bounded function  $f \in C^{\infty}(\Omega)$  such that 
$$\|g - f\|_{L^2(\Omega)}^2 \leq \frac{\epsilon}{4}.$$
 Further, for a function  $f \in C^{\infty}(\Omega)$  given  $\epsilon$  we find  $i_0 > 0$  such that 
$$\int_{\Omega_i} |f(y)|^2 dy \leq \frac{\epsilon}{4}$$
 for all  $i \geq i_0$ . Now, given  $\epsilon$  we first choose  $f \in C^{\infty}$  such that (ref{eq2}) is satisfied. Then, we choose  $i_0$  such that (ref{eq3}) is true. This yields 
$$\int_{\Omega_i} |g|^2 dy = \int_{\Omega_i} |g-f+f|^2 dy \leq 2 \int_{\Omega_i} |g-f|^2 dy + \int_{\Omega_i} |f|^2 dy \leq 2 \left( \frac{\epsilon}{4} \right) + \frac{\epsilon}{4} = \epsilon$$
 for  $i \geq i_0$ .  $\square$

## another statement

$A: X \rightarrow Y$  compact,  $\varphi_n$  bounded sequence in  $X$ , then  $\psi_n := A\varphi_n$  has a convergent subsequence, which we relabel to use index  $n$  again.

$\psi_n \rightarrow \psi_{\ast} \in Y$ . If  $\psi_{\ast} \in R(A)$ , then there exists  $\varphi_{\ast} \in X$  such that  $A\varphi_{\ast} = \psi_{\ast}$ . Defining  $\tilde{\varphi}_n := \varphi_n - \varphi_{\ast}$  we obtain a sequence such that  $A\tilde{\varphi}_n \rightarrow 0, \quad n \rightarrow \infty.$

[misc2feigt](#)

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